Designing a Unit on Quadratic Equations using the TfU Framework: A Work in Progress!

by Molly Barrett – Head-Royce School

Original Problem:

Before this year, I taught algebra to two sections of 8th graders each of the previous three years. We usually started the unit on quadratic equations (mainly graphing and solving) around the end of March or the beginning of April. I always felt rushed to get through the curriculum and never felt like most students really had the big picture of what we were doing. I also did not feel that the main ideas of the unit were clear to the students. This is mainly because I had not carefully considered them myself, nor had I explicitly discussed the main overarching ideas.

After attending the Project Zero workshop in June 2003, I joined the PZ Fellows group for 2003-4. I decided to focus on the quadratic functions unit and how I could be more deliberate both in what I taught and how it was taught in order to foster better understanding among my students.

Background:

Project Zero started in 1967 as a group to research the development of learning processes at Harvard's Graduate School of Education. More recently, the Teaching for Understanding project spent five years developing and testing a pedagogy of understanding. One result, the Teaching for Understanding (TfU) framework, is a structure for planning a curriculum to promote in-depth understanding.

The main components of the framework are as follows:

- Throughlines: A few broad, overarching questions or statements that thread together a year's curriculum.
- Generative Topics: A focus for the unit of study that will interest both teacher and student.
- Understanding goals: The most important understandings necessary for the unit. These should be closely related to the throughlines for the course.
- Performances of understanding: Activities that allow students to demonstrate and build upon their understanding. These should be directly related to the understanding goals.
- Ongoing Assessment: Assessments should be varied in style and in the person giving the feedback (student, teacher). It should focus on the understanding goals for the unit, and students should be clear about what specifically constitutes "good" work.

Goal:

One of the main ideas that stuck with me from the PZ conference was the notion of being as deliberate in identifying what you teach as in determining how to teach it. Previously, I taught the sections in our curriculum that were deemed most important without much explicit explanation of the main overarching ideas. Therefore, my main goal for this project was to redesign the quadratic functions unit using the TfU framework. To do this, I began by identifying the understanding goals and then decided to design appropriate performances of understanding and assessments. I wanted to make these public to my students and to assess their usefulness to students at the end of the unit.

Tools:

In addition to the TfU materials from the PZ conference, I used the CCDT (Collaborative Curriculum Design Tool) site online on the ALPS website (http://learnweb.harvard.edu/ccdt/index.cfm) to help arrange

my ideas. I engaged the help of the other 8th grade algebra teacher, the 9th grade geometry teachers, and the head of the Math Department to review the unit from a mathematical perspective. I also asked the academic dean to look at it from a broader point of view. Finally, Lynley Harold and Ann Jaquith provided invaluable support, advice, and ideas at our PZ meetings.

Process:

Once I had my goal in mind, I decided to firm up some throughlines I'd written last spring for our algebra class. These would help me link the understanding goals back to the main threads of the year in algebra, and would anchor my newly designed unit firmly in our curriculum. The throughlines on which I decided are listed below:

1. Using your information: How can I use what I know to find out what I don't know?
2. Modeling: How can equations and graphs help me model my world?
3. Communication: What does a sufficient answer look like?
4. Three Forms: A table, graph, and equation all show the same relationship in a different way.

I proceeded to ask the other 8th grade algebra teacher and the high school math teachers for their opinions on the most important understandings from Algebra I. I merged the responses to get a preliminary draft of the understanding goals. At this point, I also attempted to identify performances of understanding and assessments that would promote the understanding goals.

At our March PZ fellows meeting, Lynley and Ann helped me to focus the understanding goals (UGs) to the ones I now have. There were several concerns about the original draft. First, there were too many (14) understanding goals for one unit. There was also a fair amount of overlap. For example, I found that several of my understanding goals could really be simplified into one: "Students will understand and be able to use appropriately the vocabulary of the unit." After further consultation with the other algebra teacher at Head-Royce, I realized that one or two UGs were not important enough to stay on the list.

Lynley and Ann also helped me to clearly match the UGs to the throughlines of our course, and to carefully consider if the Performances of Understanding were going to promote the understanding goals.

Product:

My most recent version of the unit in the TfU framework is attached. At this point, I feel that the understanding goals are appropriate and could be useful to students to help focus their work for the unit. The other parts of the framework (PUs and assessment) are not as well developed and still need quite a bit of attention. My students all have a copy of the throughlines for the algebra course as well as the understanding goals for this unit (attached).

In creating this document, I found it invaluable to focus on the main ideas of the unit. I realized I often get bogged down in the details of skills and day-to-day assignments and forget to relate them back to the big picture. Though I often relate skills and concepts to real world situations, I have really appreciated the structure that the throughlines and TfU framework provide.

Assessment:

My project is far from finished! I had hoped to really put this to use during the quadratic equations unit this school year, but the unit just ended yesterday and I did not do much of what I intended. My students received a copy of the throughlines for the class and the understanding goals for the unit. However, I did not refer back to them as much as I wanted to in order to thread together the different lessons, activities, and concepts we covered. Beginning next year with throughlines will be a bit easier, though I will still need to make sure to refer to them often. They are posted on the board in my classroom but I'm not yet in the habit of actively using them.

My students and I went through the UGs at the start of the unit but didn't come back to them until this week before the test. I would have liked to assess if they were useful to my students, but did not feel like I even spent enough time referring to them to get a valid assessment of their utility.

I found this project quite valuable. The TfU framework is easy to use and to apply, with or without the jargon. It has provided a structure for me to think more broadly about the curriculum I teach, which has

in turn made the way I run my class more deliberately aimed at certain understandings. Both the process and the product have been beneficial to me and my teaching, which will in turn be in beneficial to my students.

What's next?
1) To post and refer to the throughlines from the beginning of the year to the end.
2) To rework the assessment (both formal and informal) of the quadratic functions unit to assess student understanding of the UGs as well as the utility of the UGs to the students.
3) Hopefully, to go through this process for other units we cover in 8th grade. My sense is that the determination of the UGs will be somewhat less challenging, though just as important.

ccdt
collaborative curriculum design tool
Quadratic Functions

author(s): Molly Barrett
grade: 8
subject: Algebra
description: A unit on quadratic functions based on the TfU framework. Particular emphasis was placed on identifying the understanding goals.

THROUGHLINES
1. What you know and what you don't: How can I use what I know to find out what I don't know?
2. Modeling: How can equations and graphs help me model my world?
3. Communication: What does a sufficient answer look like?
4. Table, graph, equation: A table, graph, and equation all show the same data and relationship in a different way.

GENERATIVE TOPICS
1. Parabolas: Snowboards, Skis, Satellite Dishes, and Water Rockets: Students will learn how common parabolas are in their world, and how they can use quadratic equations to model them.

UNIT LEVEL UNDERSTANDING GOALS
1. Graph and equation: Students will understand the relationship between the graph of a linear function and its equation.
2. Vocabulary: Students will know and apply the vocabulary of quadratic functions appropriately.
3. QF: Students will understand the use and appropriate application of the quadratic formula in finding roots or solving quadratic equations.
4. Factor: Students will learn how to solve quadratic equations by factoring.
5. Factoring vs. the QF: Students will be able to pick an appropriate strategy for solving quadratic equations (graphing, QF, factoring, etc.).
6. Writing quadratic equations: Students will be able to write quadratic equations to model real-world data and relationships.
7. linear vs. quadratic: Students will understand the difference(s) between linear and quadratic rates of change.

PERFORMANCES OF UNDERSTANDING
1. Real world parabolas (guided): Students will find an example of a real world parabola and overlay a grid on it to determine a quadratic equation that bests fits their data.
2. Identifying linear vs. quadratic data (guided): Students will identify data sets as demonstrating linear relationships or quadratic relationships.
3. Data gathering and analysis (guided): Students will use calculator based rangers and TI-83 graphing calculators to collect both linear and quadratic data to determine equations that model real-world relationships.
4. Unit test: Students will complete a unit test based on the understanding goals to demonstrate their knowledge of quadratic functions.

ONGOING ASSESSMENT
1. Think Pair Share questions (peer): Students will think-pair-share in table groups to discuss conceptual questions.
2. homework quiz (self):
3. Compare vs. contrast (informal): Students will compare and contrast linear, quadratic, and exponential rate of change.

Based on the graphic organizer presented in the Teaching for Understanding Guide by Blythe & Associates.

Understanding Goals – Chapters 9 and 10

- Students will know and apply the vocabulary of quadratic functions appropriately.**
- Students will understand the use and appropriate application of the quadratic formula in finding roots or solving quadratic equations.
- Students will learn how to solve quadratic equations by factoring.
- Students will be able to pick an appropriate strategy for solving quadratic equations (graphing, using the quadratic formula, factoring, etc.).
- Students will understand the difference(s) between linear and quadratic rates of change.
- Students will understand the relationship between the graph of a linear function and its equation.
- Students will be able to write quadratic equations to model real-world data and relationships.

**(the vocab in italics you've seen before)
CHAPTER 9 VOCAB: SQUARE ROOT, POSITIVE SQUARE ROOT, NEGATIVE SQUARE ROOT, RADICAND, PERFECT SQUARE, IRRATIONAL NUMBER, RADICAL EXPRESSION, QUADRATIC EQUATION IN STANDARD FORM, LEADING COEFFICIENT, SIMPLEST FORM OF A RADICAL EXPRESSION, VERTEX, ROOTS, AXIS OF SYMMETRY, QUADRATIC FORMULA
CHAPTER 10 VOCAB: POLYNOMIAL, MONOMIAL, BINOMIAL, TRINOMIAL, FOIL, FACTORED FORM, ZERO –PRODUCT PROPERTY, FACTOR A QUADRATIC EXPRESSION, PRIME FACTOR, FACTORING A QUADRATIC EQUATION COMPLETELY